RECOGNISING ACHIEVEMENT

ADVANCED GCE **4756/01**

MATHEMATICS (MEI)

Further Methods for Advanced Mathematics (FP2)

WEDNESDAY 9 JANUARY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

Section A (54 marks)

Answer all the questions

1 **(a)** Fig. 1 shows the curve with polar equation $r = a(1 - \cos 2\theta)$ for $0 \leqslant \theta \leqslant \pi$, where a is a positive constant.

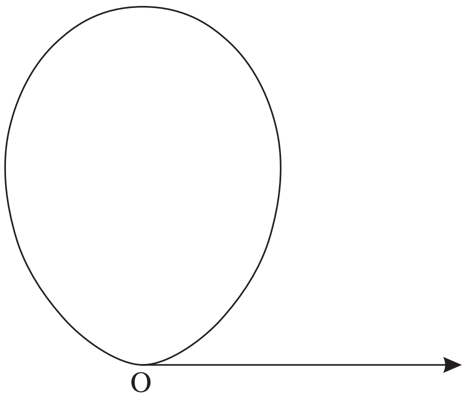

Fig. 1

Find the area of the region enclosed by the curve. [7]

(b) **(i)** Given that $\mathrm{f}(x) = \arctan(\sqrt{3} + x)$, find $\mathrm{f}'(x)$ and $\mathrm{f}''(x)$. [4]

(ii) Hence find the Maclaurin series for $\arctan(\sqrt{3} + x)$, as far as the term in x^2. [4]

(iii) Hence show that, if h is small, $\displaystyle\int_{-h}^{h} x \arctan(\sqrt{3} + x)\,\mathrm{d}x \approx \tfrac{1}{6}h^3$. [3]

2 **(a)** Find the 4th roots of 16j, in the form $r\mathrm{e}^{\mathrm{j}\theta}$ where $r > 0$ and $-\pi < \theta \leqslant \pi$. Illustrate the 4th roots on an Argand diagram. [6]

(b) **(i)** Show that $(1 - 2\mathrm{e}^{\mathrm{j}\theta})(1 - 2\mathrm{e}^{-\mathrm{j}\theta}) = 5 - 4\cos\theta$. [3]

Series C and S are defined by

$$C = 2\cos\theta + 4\cos 2\theta + 8\cos 3\theta + \ldots + 2^n \cos n\theta,$$
$$S = 2\sin\theta + 4\sin 2\theta + 8\sin 3\theta + \ldots + 2^n \sin n\theta.$$

(ii) Show that $C = \dfrac{2\cos\theta - 4 - 2^{n+1}\cos(n+1)\theta + 2^{n+2}\cos n\theta}{5 - 4\cos\theta}$, and find a similar expression for S. [9]

3 You are given the matrix $\mathbf{M} = \begin{pmatrix} 7 & 3 \\ -4 & -1 \end{pmatrix}$.

(i) Find the eigenvalues, and corresponding eigenvectors, of the matrix **M**. [8]

(ii) Write down a matrix **P** and a diagonal matrix **D** such that $\mathbf{P}^{-1}\mathbf{M}\mathbf{P} = \mathbf{D}$. [2]

(iii) Given that $\mathbf{M}^n = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, show that $a = -\frac{1}{2} + \frac{3}{2} \times 5^n$, and find similar expressions for b, c and d. [8]

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(i)** Given that $k \geqslant 1$ and $\cosh x = k$, show that $x = \pm \ln\left(k + \sqrt{k^2 - 1}\right)$. [5]

(ii) Find $\displaystyle\int_1^2 \frac{1}{\sqrt{4x^2 - 1}}\,dx$, giving the answer in an exact logarithmic form. [5]

(iii) Solve the equation $6\sinh x - \sinh 2x = 0$, giving the answers in an exact form, using logarithms where appropriate. [4]

(iv) Show that there is no point on the curve $y = 6\sinh x - \sinh 2x$ at which the gradient is 5. [4]

[Question 5 is printed overleaf.]

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 A curve has parametric equations $x = \dfrac{t^2}{1+t^2}$, $y = t^3 - \lambda t$, where λ is a constant.

(i) Use your calculator to obtain a sketch of the curve in each of the cases

$$\lambda = -1, \quad \lambda = 0 \quad \text{and} \quad \lambda = 1.$$

Name any special features of these curves. [5]

(ii) By considering the value of x when t is large, write down the equation of the asymptote. [1]

For the remainder of this question, assume that λ is positive.

(iii) Find, in terms of λ, the coordinates of the point where the curve intersects itself. [3]

(iv) Show that the two points on the curve where the tangent is parallel to the x-axis have coordinates

$$\left(\frac{\lambda}{3+\lambda}, \pm\sqrt{\frac{4\lambda^3}{27}}\right).$$ [4]

Fig. 5 shows a curve which intersects itself at the point (2, 0) and has asymptote $x = 8$. The stationary points A and B have y-coordinates 2 and -2.

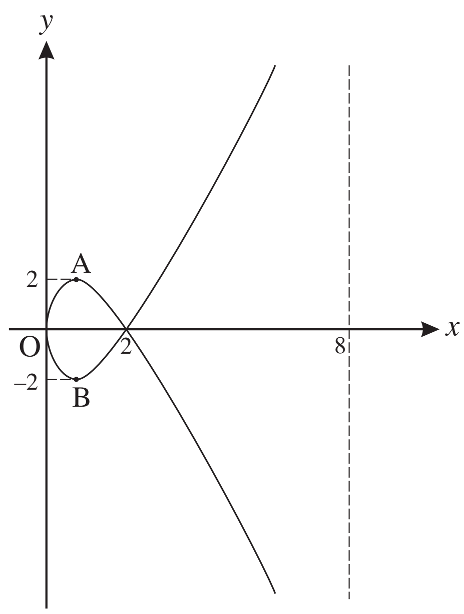

Fig. 5

(v) For the curve sketched in Fig. 5, find parametric equations of the form $x = \dfrac{at^2}{1+t^2}$, $y = b(t^3 - \lambda t)$, where a, λ and b are to be determined. [5]

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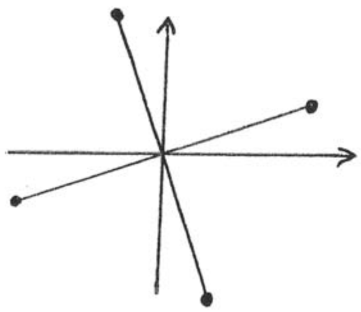
1(a)	Area is $\int_0^{\pi} \frac{1}{2}a^2(1-\cos 2\theta)^2\,d\theta$ $= \int_0^{\pi} \frac{1}{2}a^2\left(1-2\cos 2\theta + \frac{1}{2}(1+\cos 4\theta)\right)d\theta$ $= \frac{1}{2}a^2\left[\frac{3}{2}\theta - \sin 2\theta + \frac{1}{8}\sin 4\theta\right]_0^{\pi}$ $= \frac{3}{4}\pi a^2$	M1 A1 B1 B1B1B1 ft A1 **7**	For $\int (1-\cos 2\theta)^2\,d\theta$ Correct integral expression including limits (may be implied by later work) For $\cos^2 2\theta = \frac{1}{2}(1+\cos 4\theta)$ Integrating $a + b\cos 2\theta + c\cos 4\theta$ *[Max B2 if answer incorrect and no mark has previously been lost]*
(b)(i)	$f'(x) = \dfrac{1}{1+(\sqrt{3}+x)^2}$ $f''(x) = \dfrac{-2(\sqrt{3}+x)}{\left(1+(\sqrt{3}+x)^2\right)^2}$	M1 A1 M1 A1 **4**	Applying $\dfrac{d}{du}\arctan u = \dfrac{1}{1+u^2}$ or $\dfrac{dy}{dx} = \dfrac{1}{\sec^2 y}$ Applying chain (or quotient) rule
(ii)	$f(0) = \frac{1}{3}\pi$ $f'(0) = \frac{1}{4},\quad f''(0) = -\frac{1}{8}\sqrt{3}$ $\arctan(\sqrt{3}+x) = \frac{1}{3}\pi + \frac{1}{4}x - \frac{1}{16}\sqrt{3}\,x^2 + \ldots$	B1 M1 A1A1 ft **4**	Stated; or appearing in series *Accept* 1.05 Evaluating $f'(0)$ or $f''(0)$ For $\frac{1}{4}x$ and $-\frac{1}{16}\sqrt{3}\,x^2$ *ft provided coefficients are non-zero*
(iii)	$\int_{-h}^{h} (\frac{1}{3}\pi x + \frac{1}{4}x^2 - \frac{1}{16}\sqrt{3}\,x^3 + \ldots)\,dx$ $= \left[\frac{1}{6}\pi x^2 + \frac{1}{12}x^3 - \frac{1}{64}\sqrt{3}\,x^4 + \ldots\right]_{-h}^{h}$ $\approx (\frac{1}{6}\pi h^2 + \frac{1}{12}h^3 - \frac{1}{64}\sqrt{3}\,h^4)$ $-(\frac{1}{6}\pi h^2 - \frac{1}{12}h^3 - \frac{1}{64}\sqrt{3}\,h^4)$ $= \frac{1}{6}h^3$	M1 A1 ft A1 ag **3**	Integrating (award if *x* is missed) for $\frac{1}{12}x^3$ Allow ft from $a + \frac{1}{4}x + cx^2$ provided that $a \neq 0$ Condone a proof which neglects h^4

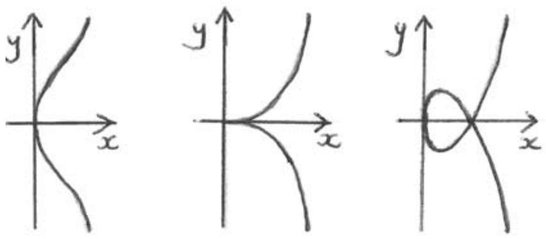
2(a)	4th roots of $16\mathrm{j} = 16\,\mathrm{e}^{\frac{1}{2}\pi\mathrm{j}}$ are $r\,\mathrm{e}^{\mathrm{j}\theta}$ where $r = 2$ $\theta = \frac{1}{8}\pi$ $\theta = \dfrac{\pi}{8} + \dfrac{2k\pi}{4}$ $\theta = -\frac{7}{8}\pi,\ -\frac{3}{8}\pi,\ \frac{5}{8}\pi$ 	B1 B1 M1 A1 M1 A1 **6**	*Accept* $16^{\frac{1}{4}}$ Implied by at least two correct (ft) further values or stating $k = -2,\ -1,\ (0),\ 1$ Points at vertices of a square centre O *or* 3 correct points (ft) *or* 1 point in each quadrant
(b)(i)	$(1-2\,\mathrm{e}^{\mathrm{j}\theta})(1-2\,\mathrm{e}^{-\mathrm{j}\theta}) = 1-2\,\mathrm{e}^{\mathrm{j}\theta}-2\,\mathrm{e}^{-\mathrm{j}\theta}+4$ $= 5-2(\mathrm{e}^{\mathrm{j}\theta}+\mathrm{e}^{-\mathrm{j}\theta})$ $= 5-4\cos\theta$	M1 A1 A1 ag **3**	For $\mathrm{e}^{\mathrm{j}\theta}\mathrm{e}^{-\mathrm{j}\theta} = 1$
	OR $(1-2\cos\theta-2\mathrm{j}\sin\theta)(1-2\cos\theta+2\mathrm{j}\sin\theta)$ M1 $= (1-2\cos\theta)^2+4\sin^2\theta$ A1 $= 1-4\cos\theta+4(\cos^2\theta+\sin^2\theta)$ $= 5-4\cos\theta$ A1		
(ii)	$C+\mathrm{j}S = 2\,\mathrm{e}^{\mathrm{j}\theta}+4\,\mathrm{e}^{2\mathrm{j}\theta}+8\,\mathrm{e}^{3\mathrm{j}\theta}+\ldots+2^n\,\mathrm{e}^{n\mathrm{j}\theta}$ $= \dfrac{2\,\mathrm{e}^{\mathrm{j}\theta}\left(1-(2\,\mathrm{e}^{\mathrm{j}\theta})^n\right)}{1-2\,\mathrm{e}^{\mathrm{j}\theta}}$ $= \dfrac{2\,\mathrm{e}^{\mathrm{j}\theta}(1-2^n\,\mathrm{e}^{n\mathrm{j}\theta})(1-2\,\mathrm{e}^{-\mathrm{j}\theta})}{(1-2\,\mathrm{e}^{\mathrm{j}\theta})(1-2\,\mathrm{e}^{-\mathrm{j}\theta})}$ $= \dfrac{2\,\mathrm{e}^{\mathrm{j}\theta}-4-2^{n+1}\,\mathrm{e}^{(n+1)\mathrm{j}\theta}+2^{n+2}\,\mathrm{e}^{n\mathrm{j}\theta}}{5-4\cos\theta}$ $C = \dfrac{2\cos\theta-4-2^{n+1}\cos(n+1)\theta+2^{n+2}\cos n\theta}{5-4\cos\theta}$ $S = \dfrac{2\sin\theta-2^{n+1}\sin(n+1)\theta+2^{n+2}\sin n\theta}{5-4\cos\theta}$	M1 M1 A1 M1 A2 M1 A1 ag A1 **9**	Obtaining a geometric series Summing (M0 for sum to infinity) Give A1 for two correct terms in numerator Equating real (or imaginary) parts

3 (i)	Characteristic equation is $(7-\lambda)(-1-\lambda)+12=0$ $\lambda^2-6\lambda+5=0$ $\lambda=1,\ 5$ When $\lambda=1$, $\begin{pmatrix}7 & 3\\ -4 & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\\ y\end{pmatrix}$ $7x+3y=x$ $-4x-y=y$ $y=-2x$, eigenvector is $\begin{pmatrix}1\\ -2\end{pmatrix}$ When $\lambda=5$, $\begin{pmatrix}7 & 3\\ -4 & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=5\begin{pmatrix}x\\ y\end{pmatrix}$ $7x+3y=5x$ $-4x-y=5y$ $y=-\frac{2}{3}x$, eigenvector is $\begin{pmatrix}3\\ -2\end{pmatrix}$	M1 A1A1 M1 M1 A1 M1 A1 **8**	or $\begin{pmatrix}6 & 3\\ -4 & -2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$ *can be awarded for either eigenvalue* Equation relating *x* and *y* or any (non-zero) multiple *SR* $(\mathbf{M}-\lambda\mathbf{I})\mathbf{x}=\lambda\mathbf{x}$ can earn M1A1A1M0M1A0M1A0
(ii)	$\mathbf{P}=\begin{pmatrix}1 & 3\\ -2 & -2\end{pmatrix}$ $\mathbf{D}=\begin{pmatrix}1 & 0\\ 0 & 5\end{pmatrix}$	B1 ft B1 ft **2**	B0 if **P** is singular For B2, the order must be consistent

(iii)	$\mathbf{M} = \mathbf{P}\,\mathbf{D}\,\mathbf{P}^{-1}$	M1	*May be implied*
	$\mathbf{M}^n = \mathbf{P}\,\mathbf{D}^n\,\mathbf{P}^{-1}$	M1	
	$= \mathbf{P}\begin{pmatrix} 1 & 0 \\ 0 & 5^n \end{pmatrix}\mathbf{P}^{-1}$	A1 ft	*Dependent on M1M1*
	$= \begin{pmatrix} 1 & 3 \\ -2 & -2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 5^n \end{pmatrix}\frac{1}{4}\begin{pmatrix} -2 & -3 \\ 2 & 1 \end{pmatrix}$	B1 ft	For $\mathbf{P}^{-1}$
	$= \begin{pmatrix} 1 & 3\times 5^n \\ -2 & -2\times 5^n \end{pmatrix}\frac{1}{4}\begin{pmatrix} -2 & -3 \\ 2 & 1 \end{pmatrix}$		or $\begin{pmatrix} 1 & 3 \\ -2 & -2 \end{pmatrix}\frac{1}{4}\begin{pmatrix} -2 & -3 \\ 2\times 5^n & 5^n \end{pmatrix}$
	$= \frac{1}{4}\begin{pmatrix} -2+6\times 5^n & -3+3\times 5^n \\ 4-4\times 5^n & 6-2\times 5^n \end{pmatrix}$	M1	Obtaining at least one element in a product of three matrices
	$a = -\frac{1}{2}+\frac{3}{2}\times 5^n$	A1 ag	
	$b = -\frac{3}{4}+\frac{3}{4}\times 5^n$ $c = 1-5^n$ $\quad d = \frac{3}{2}-\frac{1}{2}\times 5^n$	A2 **8**	Give A1 for one of *b, c, d* correct *SR* If $\mathbf{M}^n = \mathbf{P}^{-1}\,\mathbf{D}^n\,\mathbf{P}$ is used, max marks are M0M1A0B1M1A0A1 (*d* should be correct) *SR* If their **P** is singular, max marks are M1M1A1B0M0

4 (i)	$\frac{1}{2}(e^x + e^{-x}) = k$ $e^{2x} - 2k\,e^x + 1 = 0$ $e^x = \dfrac{2k \pm \sqrt{4k^2-4}}{2} = k \pm \sqrt{k^2-1}$ $x = \ln(k+\sqrt{k^2-1})$ or $\ln(k-\sqrt{k^2-1})$ $(k+\sqrt{k^2-1})(k-\sqrt{k^2-1}) = k^2-(k^2-1) = 1$ $\ln(k-\sqrt{k^2-1}) = \ln(\dfrac{1}{k+\sqrt{k^2-1}}) = -\ln(k+\sqrt{k^2-1})$ $x = \pm\ln(k+\sqrt{k^2-1})$	M1 M1 A1 M1 A1 ag **5**	or $\cosh x + \sinh x = e^x$ or $k \pm \sqrt{k^2-1} = e^x$ One value sufficient or $\cosh x$ is an even function *(or equivalent)*
(ii)	$\displaystyle\int_1^2 \frac{1}{\sqrt{4x^2-1}}\,dx = \left[\frac{1}{2}\,\text{arcosh}\,2x\right]_1^2$ $= \frac{1}{2}(\text{arcosh}\,4 - \text{arcosh}\,2)$ $= \frac{1}{2}\left(\ln(4+\sqrt{15}) - \ln(2+\sqrt{3})\right)$	M1 A1 A1 M1 A1 **5**	For arcosh *or* $\ln(\lambda x + \sqrt{\lambda^2 x^2 - ...})$ or any cosh substitution For $\text{arcosh}\,2x$ or $2x = \cosh u$ or $\ln(2x+\sqrt{4x^2-1})$ or $\ln(x+\sqrt{x^2-\frac{1}{4}})$ For $\frac{1}{2}$ or $\int \frac{1}{2}\,du$ Exact numerical logarithmic form
(iii)	$6\sinh x - 2\sinh x\cosh x = 0$ $\cosh x = 3$ (or $\sinh x = 0$) $x = 0$ $x = \pm\ln(3+\sqrt{8})$	M1 M1 B1 A1 **4**	 Obtaining a value for $\cosh x$ or $x = \ln(3 \pm \sqrt{8})$
	OR $e^{4x} - 6e^{3x} + 6e^x - 1 = 0$ $(e^{2x}-1)(e^{2x}-6e^x+1) = 0$ M2 $x = 0$ B1 $x = \ln(3\pm\sqrt{8})$ A1		or $(e^x - e^{-x})(e^x + e^{-x} - 6) = 0$
(iv)	$\dfrac{dy}{dx} = 6\cosh x - 2\cosh 2x$ If $\dfrac{dy}{dx} = 5$ then $6\cosh x - 2(2\cosh^2 x - 1) = 5$ $4\cosh^2 x - 6\cosh x + 3 = 0$ Discriminant $D = 6^2 - 4\times 4\times 3 = -12$ Since $D < 0$ there are no solutions	B1 M1 M1 A1 **4**	 Using $\cosh 2x = 2\cosh^2 x - 1$ Considering *D*, or completing square, or considering turning point

	OR Gradient $g = 6\cosh x - 2\cosh 2x$ B1 $g' = 6\sinh x - 4\sinh 2x = 2\sinh x(3 - 4\cosh x)$ $= 0$ when $x = 0$ (only) M1 $g'' = 6\cosh x - 8\cosh 2x = -2$ when $x = 0$ M1 Max value $g = 4$ when $x = 0$ So *g* is never equal to 5 A1		Final A1 requires a complete proof showing this is the only turning point

5 (i)	$\lambda=-1$ $\lambda=0$ $\lambda=1$  cusp loop	B1B1B1 B1B1 **5**	Two different features (cusp, loop, asymptote) correctly identified
(ii)	$x=1$	B1 **1**	
(iii)	Intersects itself when $y=0$ $t=(\pm)\sqrt{\lambda}$ $\left(\frac{\lambda}{1+\lambda},\ 0\right)$	M1 A1 A1 **3**	
(iv)	$\frac{\mathrm{d}y}{\mathrm{d}t}=3t^2-\lambda=0$ $t=\pm\sqrt{\frac{\lambda}{3}}$ $x=\frac{\lambda/3}{1+\lambda/3}=\frac{\lambda}{3+\lambda}$ $y=\pm\left((\frac{\lambda}{3})^{\frac{3}{2}}-\lambda(\frac{\lambda}{3})^{\frac{1}{2}}\right)$ $=\pm\lambda^{\frac{3}{2}}\left(\frac{1}{3\sqrt{3}}-\frac{1}{\sqrt{3}}\right)=\pm\lambda^{\frac{3}{2}}\left(-\frac{2}{3\sqrt{3}}\right)$ $=\pm\sqrt{\frac{4\lambda^3}{27}}$	M1 A1 ag M1 A1 ag **4**	One value sufficient
(v)	From asymptote, $a=8$ From intersection point, $\frac{a\lambda}{1+\lambda}=2$ $\lambda=\frac{1}{3}$ From maximum point, $b\sqrt{\frac{4\lambda^3}{27}}=2$ $b=27$	B1 M1 A1 M1 A1 **5**	

4756: Further Methods for Advanced Mathematics (FP2)

General Comments

This paper was found to be rather more straightforward than the recent past papers, and most candidates performed well on it. They displayed well-developed skills in manipulative algebra and calculus, and handled new topics such as eigenvalues and hyperbolic functions with confidence. Only a few candidates seemed to have any difficulty completing the paper in the time allowed. There was nevertheless a wide range of marks, with about one third of the candidates scoring 60 marks or more (out of 72), and about one fifth scoring fewer than half marks.
In Section A, Q1 (on calculus) and Q3 (on matrices) were answered rather better than Q2 (on complex numbers). In Section B, almost every candidate chose the hyperbolic functions option.

Comments on Individual Questions

1) This question (on polar coordinates and Maclaurin series) was generally answered well. About 20% of candidates scored full marks, and the average mark was about 14 (out of 18).

(a) Most candidates set about finding the area enclosed by the curve confidently and efficiently, and there were very many fully correct solutions. The great majority wrote down a correct integral expression for the area, although the factor ½ or the a^2 was sometimes missing. A few forgot to square the expression for r, even when they had previously written $\int \frac{1}{2} r^2 \mathrm{d}\theta$. The method for integrating $\cos^2 2\theta$ by using the double angle formula was well known, although there were many errors with signs and coefficients.

(b) In part (i), the first derivative of arctan$(\sqrt{3}+x)$ was given accurately by the great majority of candidates. Finding the second derivative proved to be slightly more of a challenge, with sign errors, and forgetting to square the denominator, occurring quite frequently.
In part (ii), most candidates demonstrated that they knew exactly how to produce a Maclaurin series; although the constant term was sometimes left as arctan $\sqrt{3}$, and the 2! was sometimes missing from the x^2 term.
In part (iii), most candidates knew what to do, and carried it out accurately. Some forgot to multiply through by x, and there were often careless errors in the integration or the evaluation. As the answer was given, this needed to be correct in every detail to earn full marks. There were a few attempts to integrate by parts.

2) About 15% of candidates scored full marks on this question (on complex numbers), and the average mark was about 11.

(a) Most candidates knew how to find the 4th roots, and very many did so correctly. Common errors were taking the modulus of 16j to be 4 instead of 16, and giving the arguments of the 4th roots as $\frac{\pi}{2}+\frac{2k\pi}{4}$ instead of $\frac{\pi}{8}+\frac{2k\pi}{4}$. It was very pleasing to see that almost all candidates took care to give the arguments in the required range. The Argand diagram was frequently drawn on separate graph paper, which was unnecessary.

(b) The identity in part (i) caused very little difficulty.
In part (ii), almost all candidates knew that they should consider C + jS, but some made no progress beyond this. The series was usually recognised to be geometric, and an attempt made to sum it, although many considered the sum to infinity instead of the sum of n terms. In the sum to n terms, r^n was quite often written as $2\mathrm{e}^{\mathrm{j}n\theta}$ instead of $2^n\mathrm{e}^{\mathrm{j}n\theta}$. Using part (i) to obtain a real denominator, and considering the real and imaginary parts, was quite well done, although the expression for S often included an extra −4 in the numerator.

3) About one third of candidates scored full marks on this question (on matrices), and the average mark was about 13. Most candidates showed a high degree of confidence and skill with matrices.
In part (i), almost every candidate understood the methods for finding eigenvalues and eigenvectors. The eigenvalues were usually found correctly, but a fairly common error with the eigenvectors was, after correctly obtaining, say, $y = -2x$, to give $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ instead of $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$.
In part (ii), most candidates knew how to find the matrices P and D, and indeed there were very few cases of candidates giving the columns of the two matrices in an inconsistent order.
Some candidates omitted part (iii) altogether, some tried to apply the Cayley-Hamilton theorem, and some tried evaluating $\mathbf{M}^2$, $\mathbf{M}^3$ and so on. However, most candidates proceeded by evaluating $\mathbf{PD}^n\mathbf{P}^{-1}$ (although some did have $\mathbf{P}^{-1}\mathbf{D}^n\mathbf{P}$) and on the whole the manipulation was carried out accurately. Failure to obtain the correct results was usually due to incorrect eigenvectors rather than errors made in this part of the question. A few candidates had the annoying habit of making corrections by writing over their previous figures, which sometimes made their work almost illegible.

4) Most candidates demonstrated competence in the handling of hyperbolic functions, but very few managed to score full marks. The average mark was about 10.
Part (i) was the hardest item on the question paper, judging by the percentage of candidates who answered it correctly. Most candidates could obtain $x = \ln(k \pm \sqrt{k^2 - 1})$, but the majority just stopped here, or boldly asserted that the given result followed from this. Only a few could show that $\ln(k - \sqrt{k^2 - 1}) = -\ln(k + \sqrt{k^2 - 1})$; almost all the successful candidates did this algebraically, by first showing that $(k - \sqrt{k^2 - 1})(k + \sqrt{k^2 - 1}) = 1$. Several candidates stated that $\cosh x$ was an even function, or drew a sketch of the graph of $y = \cosh x$, but usually did not explain how the given result follows from this. As the result is given on the question paper, a complete and convincing explanation was required in order to earn full marks.
The integral in part (ii) was very often evaluated correctly, by going either via ½ arcosh $2x$ or straight to the logarithmic form. By far the most common error was omission of the factor ½.
In part (iii), those who rewrote the equation in terms of exponentials were rarely able to deal with the resulting quartic equation. Success was usually dependent on using sinh $2x$ = 2sinh $x$cosh x, leading quickly to sinh x = 0 or cosh x = 3. It was pleasing that only a few candidates missed the solution x = 0; but it was surprising, given the presence of part (i), how many candidates gave only one

solution for cosh x = 3.
Part (iv) was quite often answered correctly, with most successful solutions rearranging $\frac{\mathrm{d}y}{\mathrm{d}x}$ = 5 as a quadratic equation in cosh x and showing that the discriminant was negative. Quite a number gave the derivative of sinh $2x$ as ½cosh $2x$ instead of 2cosh $2x$.

5) There were only seven attempts at this question (on investigation of curves). One of these was substantially correct, but most consisted of just a few fragments.